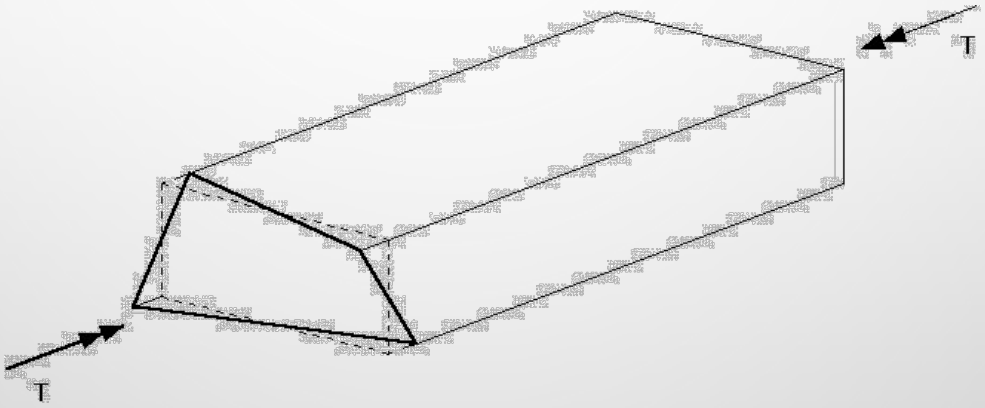
Warping Effect (7th DOF)



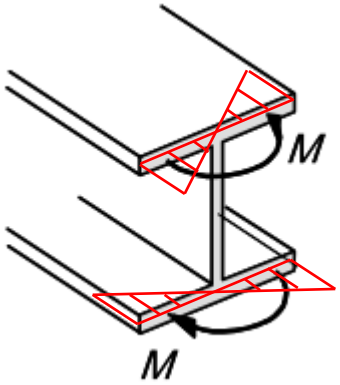
Aug. 22, 2014

Midas IT

- When bending moment occurs in a beam element,
  → Law of plane maintenance is valid.

- When shear force occurs in a beam element,
  → Law of plane maintenance is valid.

- When twisting moment occurs in a beam element,
  → Law of plane maintenance is **not** valid due to Warping.
  (excepting for pipe & solid round section)



- ✓ Torsional Rigidity of Pipe & Solid Round Section = G * I_p
- ✓ Torsional Rigidity of Other Section = 1~2% of G * I_p

Where, Ip: Polar moment of inertia, Ixx + Iyy

G: Shear modulus

CLASSIFICATION TORSION

- **Uniform Torsion**: Member is allowed to warp freely

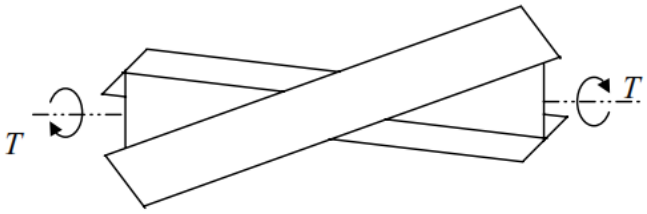
  Torque is resisted entirely by St. Venant torsional shear stress.

  St Venant Torsion, *Tsv*

- **Non-uniform Torsion**: Member is not allowed to warp freely

  Torque is resisted by St.Venant's torsional shear stress & Warping torsion

  St Venant Torsion, *Tsv* + Warping Torsion, *Tw*



Uniform Torsion (Constant Torque : Ends are free to warp)

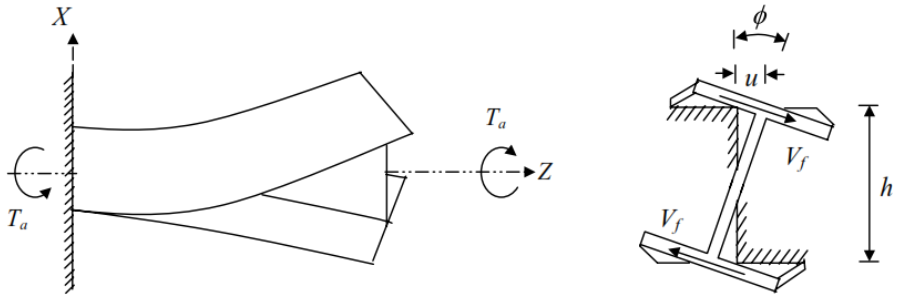
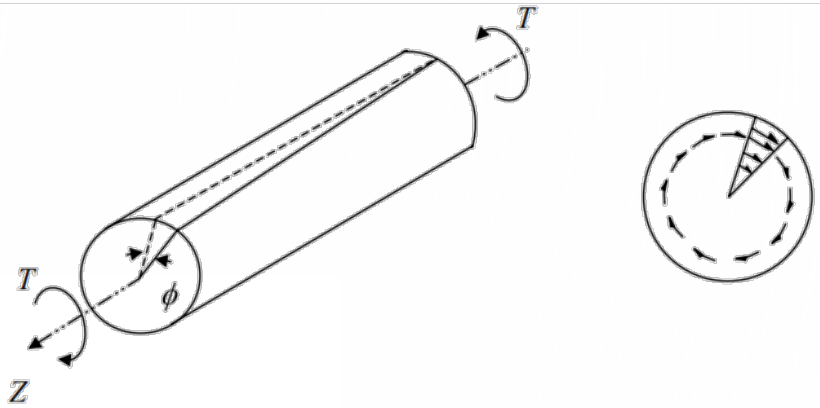


Fig. 5 Non uniform Torsion:Twisting of Non-Circular Section restrained against free warping (Constant Torque : End warping is prevented)

UNIFORM TORSION

❑ Circular Section

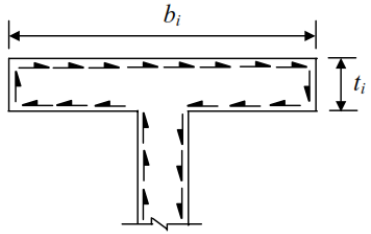
$$T_{sv} = I_p G \frac{d\phi}{dz}$$



where,
- ϕ - angle of twist
- G - modulus of rigidity
- T_{sv} - St. Venant's torsion.
- I_p - the polar moment of inertia
- z - direction along axis of the member.

❑ Non-circular Section

$$T_{sv} = JG \frac{d\phi}{dz}$$



Replace I_p by ***J*, the torsional constant**

For rectangular section, $J = C.\ bt^3$

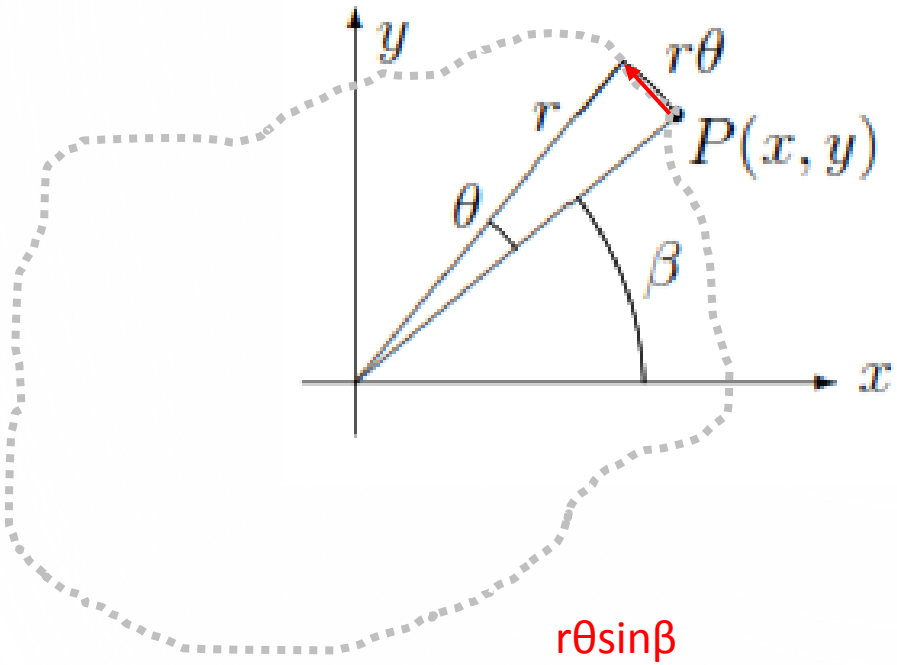
For Plate Girder, $J = \frac{1}{3}\sum_i b_i (t_i)^3$

Where, *b: breadth and thickness of the rectangle*

t: thickness of the rectangle

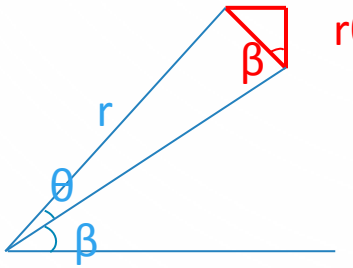
C: constant depending upon (*b/t*) *ratio and tends to 1/3 as b/t increase s*

WARPING FUNCTION



X: r cosβ
y: r sinβ

rθsinβ



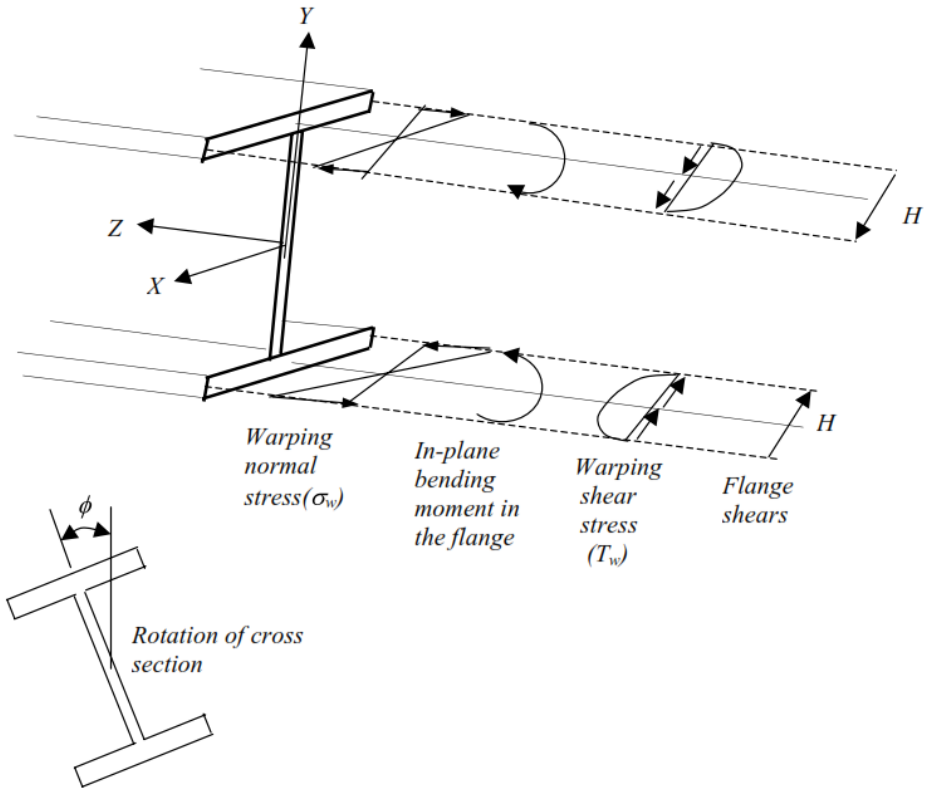
rθcosβ

- ✓ Displacement
  - ux = rθsinβ = θy
  - uy=rθcosβ = θx
  - uz = θ Φ(x,y)
- ✓ Torsional Constant

$$J = I - \int_A \nabla\phi \cdot \nabla\phi \, dA.$$

NON-UNIFORM TORSION

- ✓ Non-Uniform torsion occurs when warping deformation is constrained.
- ✓ Warping restraint causes in-plane bending deformation of the flanges in addition to twisting.
- ✓ Bending deformation is accompanied by a shear force in each flange.



NON-UNIFORM TORSION

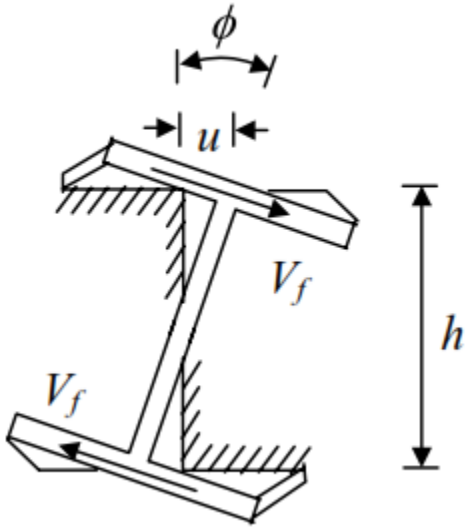
❑ Warping torsion

$$T_w = V_f . h$$

$$V_f = -\frac{dM_f}{dz} \qquad M_f = EI_f \frac{d^2u}{dz^2}$$

Curvature

$$u = \phi h / 2$$



$$M_f = \frac{EI_f h}{2} \frac{d^2\phi}{dz^2} = \frac{EI_f h}{2} \phi''$$

$$T_w = -\frac{EI_f h^2}{2} \frac{d^3\phi}{dz^3} = -\frac{EI_f h^2}{2} \phi'''$$

warping constant (Γ)

$$T_w = -E\Gamma \frac{d^3\phi}{dz^3} = -E\Gamma \phi'''$$

NON-UNIFORM TORSION

Total torsion

$$T_n = T_{sv} + T_w$$

$$T_w = -E\Gamma \frac{d^3\phi}{dz^3} = -E\Gamma\,\phi'''$$

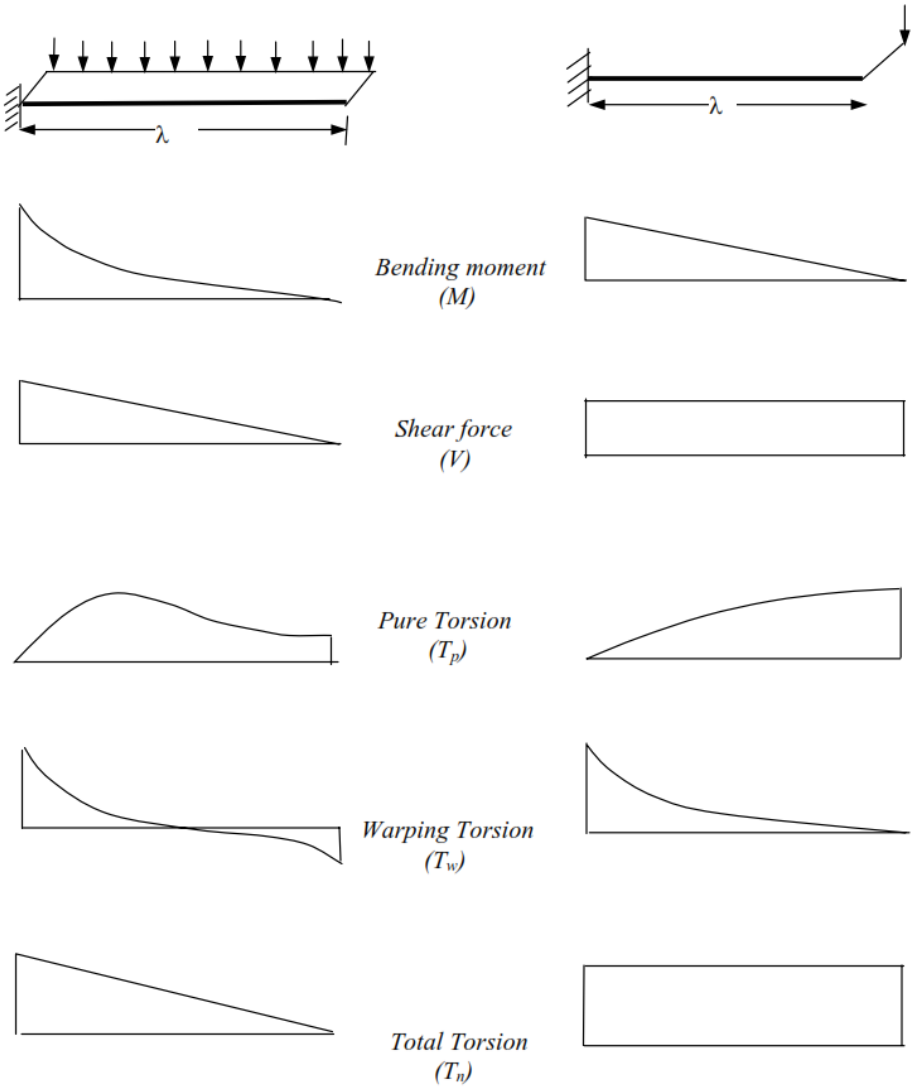
$$T_{sv} = JG\frac{d\phi}{dz}$$

Torsional Rigidity

$$T_n(z) = GJ\frac{d\phi}{dz} - E\Gamma\frac{d^3\phi}{dz^3} = GJ\phi' - E\Gamma\,\phi'''$$

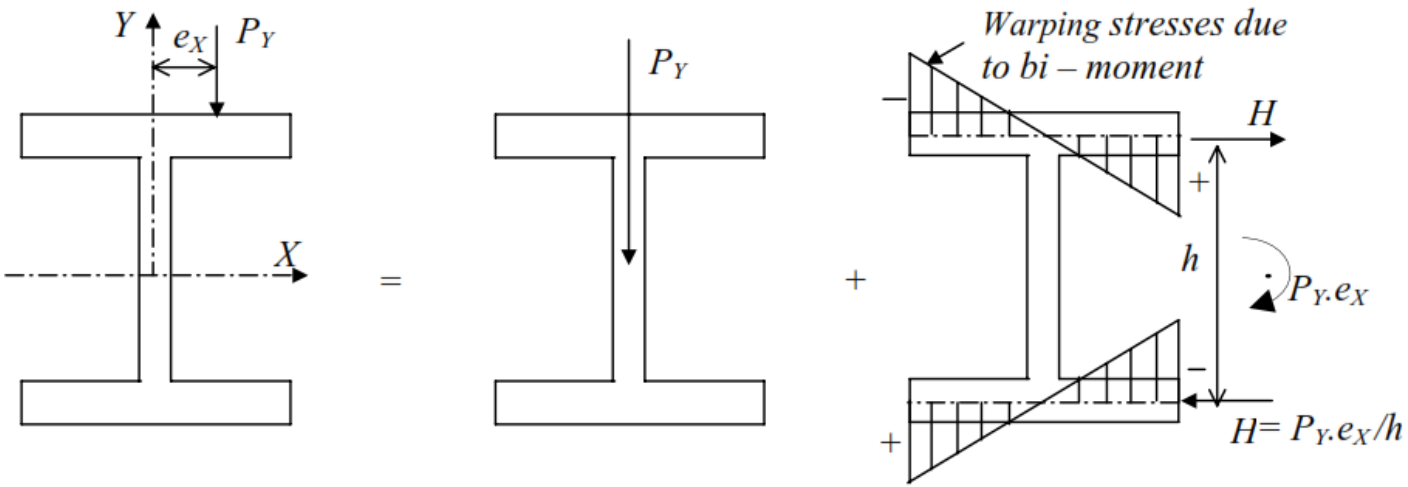
Warping Rigidity

NON-UNIFORM TORSION



BI-MOMENT METHOD

- Approximate method of torsion analysis for practical purpose



- **Bi-moment = Mf * h**

BI-MOMENT METHOD

❑ Middle Region

1. Calculate Mf using Bi-Moment or Table in the code

1. Cantilever

$\frac{L}{a} < 0.5$	$0.5 \le \frac{L}{a} \le 2.0$	$\frac{L}{a} > 2.0$
$M_{fy} = \frac{NeL}{d_f}$	$M_{fy} = \frac{Nea}{d_f} k_1$	$M_{fy} = \frac{Nea}{d_f}$
$\phi_t = 0.32 \frac{Nea}{GJ} \left[\frac{L}{a}\right]^3$	$\phi_t = \frac{Nea}{GJ} k_2$	$\phi_t = \frac{Ne}{GJ}[L - a]$
	$a = \sqrt{\frac{EI_w}{GJ}}$	

$\frac{L}{a}$	0.5	1.0	1.5	2.0
k_1	0.50	0.75	0.92	1.00
k_2	0.040	0.237	0.566	1.000

2. Simple beam

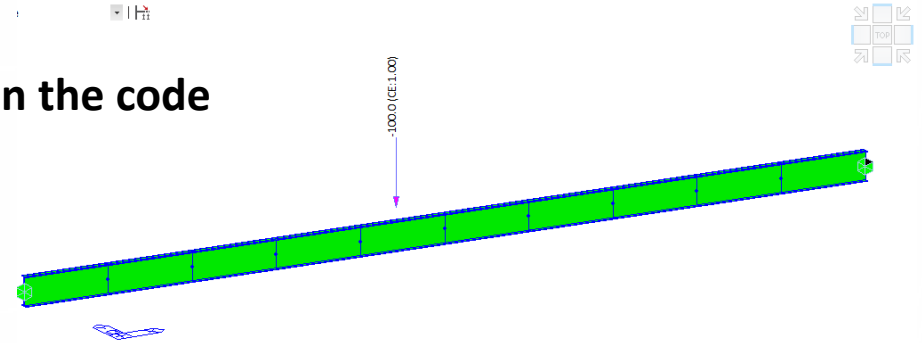
$\frac{L}{a} < 1.0$	$1.0 \le \frac{L}{a} \le 4.0$	$\frac{L}{a} > 4.0$
$M_{fy} = \frac{NeL}{4d_f}$	$M_{fy} = \frac{Nea}{2d_f} k_1$	$M_{fy} = \frac{Nea}{2d_f}$
$\phi_t = 0.32 \frac{Nea}{GJ} \left[\frac{L}{2a}\right]^3$	$\phi_t = \frac{Nea}{GJ} k_2$	$\phi_t = \frac{Ne}{GJ}\left[\frac{L}{2} - a\right]$
	$a = \sqrt{\frac{EI_w}{GJ}}$	

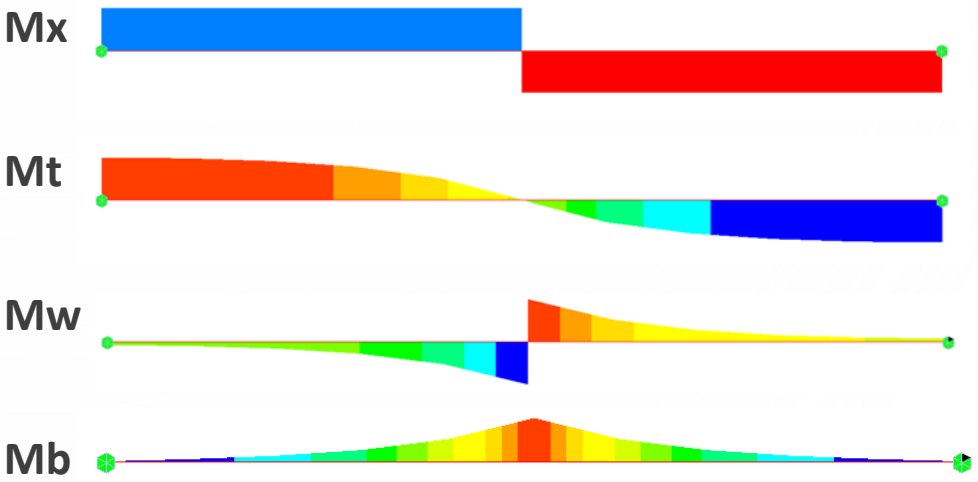
$\frac{L}{a}$	1.0	1.5	2.0	2.5	3.0	3.5	4.0
k_1	0.50	0.62	0.75	0.85	0.92	0.96	1.00
k_2	0.040	0.121	0.237	0.389	0.570	0.786	1.00

3. Continuous spans equal length, equally loaded

$\frac{L}{a} < 2.0$	$2.0 \le \frac{L}{a} \le 8.0$	$\frac{L}{a} > 8.0$
$M_{fy} = \frac{NeL}{8d_f}$	$M_{fy} = \frac{Nea}{2d_f} k_1$	$M_{fy} = \frac{Nea}{2d_f}$
$\phi_t = 0.64 \frac{Nea}{GJ} \left[\frac{L}{2a}\right]^3$	$\phi_t = \frac{Nea}{GJ} k_2$	$\phi_t = \frac{2Ne}{GJ}\left[\frac{L}{4} - a\right]$
	$a = \sqrt{\frac{EI_w}{GJ}}$	

$\frac{L}{a}$	2.0	3.0	4.0	5.0	6.0	7.0	8.0
k_1	0.50	0.62	0.75	0.85	0.92	0.96	1.00
k_2	0.040	0.121	0.237	0.387	0.570	0.786	1.000





BI-MOMENT METHOD

- Middle Region

2. Calculate Stress

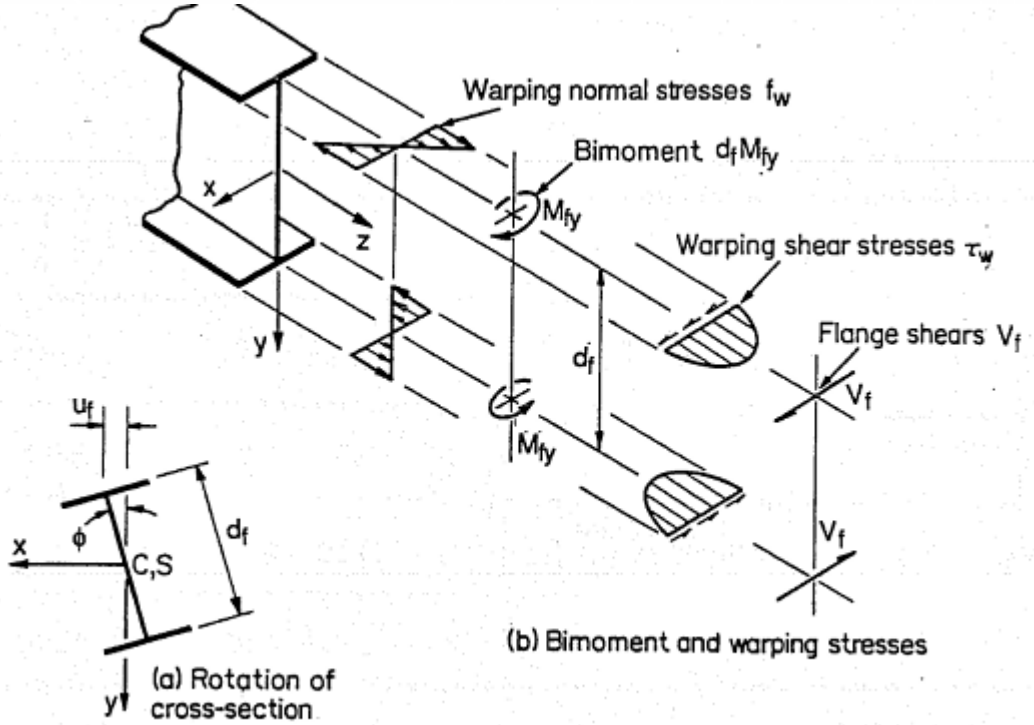
$$\sigma_w = M_f / S$$

$$V = T/d$$

$$\tau_w = 1.5V / bt$$

Normal stress = $\sigma_{bend} + \sigma_w$

Shear stress = $\tau_{bend} + \tau_w$

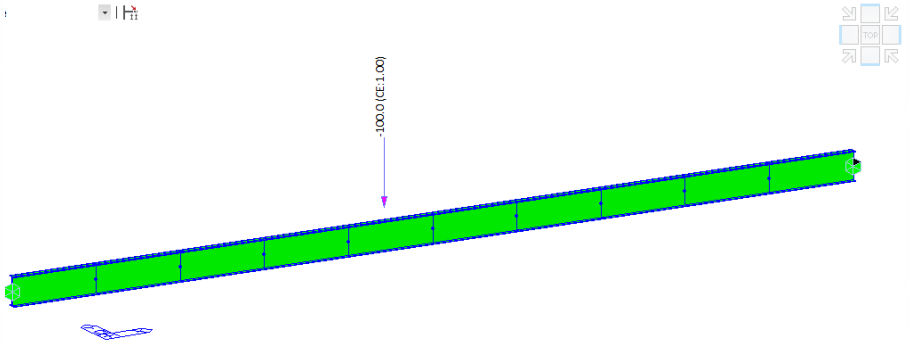


BI-MOMENT METHOD

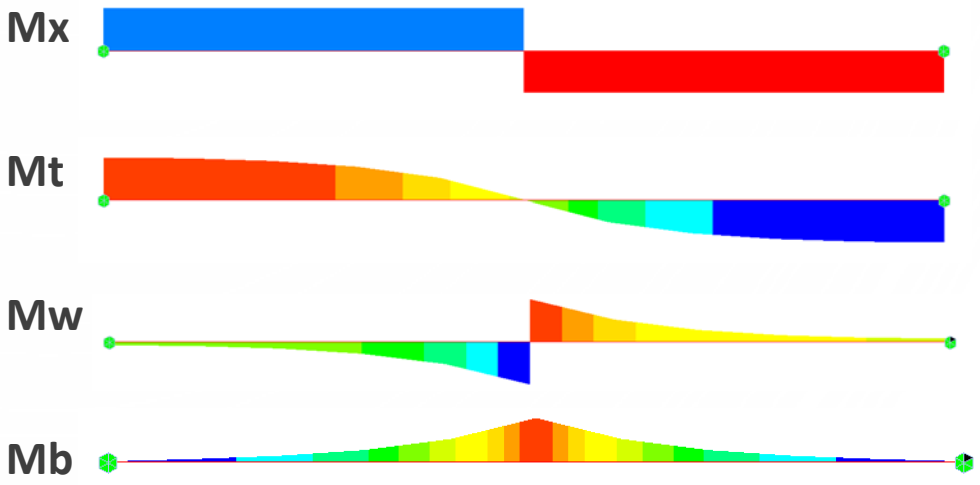
- End Region

1. Calculate Stress

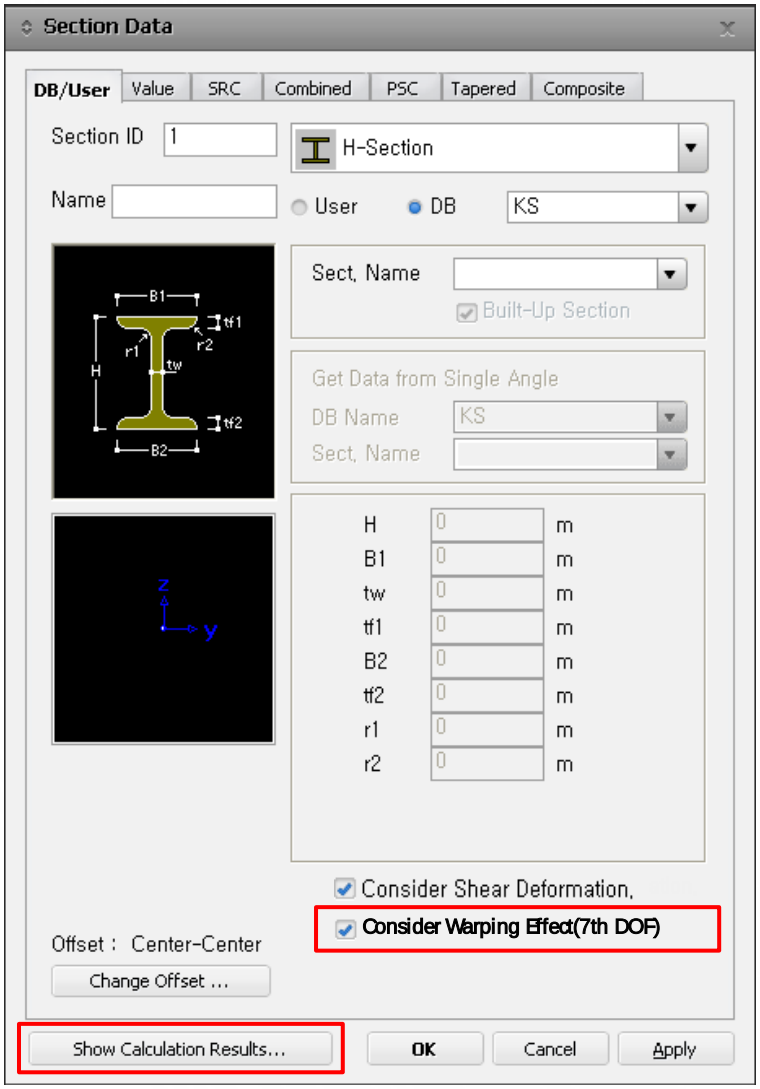
$$\tau_{sv} = Tt / J$$



Shear stress = $\tau_{bend} + \tau_{sv}$



CONSIDER WARPING EFFECT



➢ **Section Property dialog box**

- When "Consider Warping Effect(7th DOF)" option is checked on, section stiffness is calculated using **Gen Section Library (PBeamLmesh.dll).**

- When the option is not checked on, section properties and analysis results are identical to the previous version.

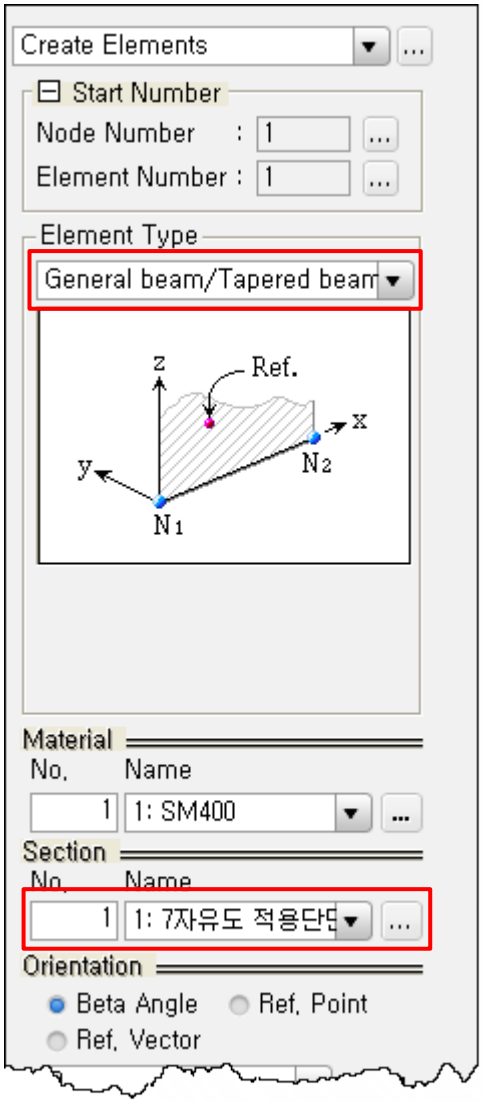
- For Tapered Section, section property is calculated using the equivalent section of i-end and j-end when **Warping Effect** is considered.

➢ **Addition of Section Properties for Warping**

- When "Consider Warping Effect(7th DOF)" option is checked on, following constants are additionally plotted.

- **Iw** : Warping Constant
- **w1, w2, w3, w4** : Warping Function at Point 1,2,3, and 4
- **γxy1, γxy2, γxy3, γxy4** : Shear Strain due to Twisting Moment
- **γxz1, γxz2, γxz3, γxz4** : Shear Strain due to Twisting Moment

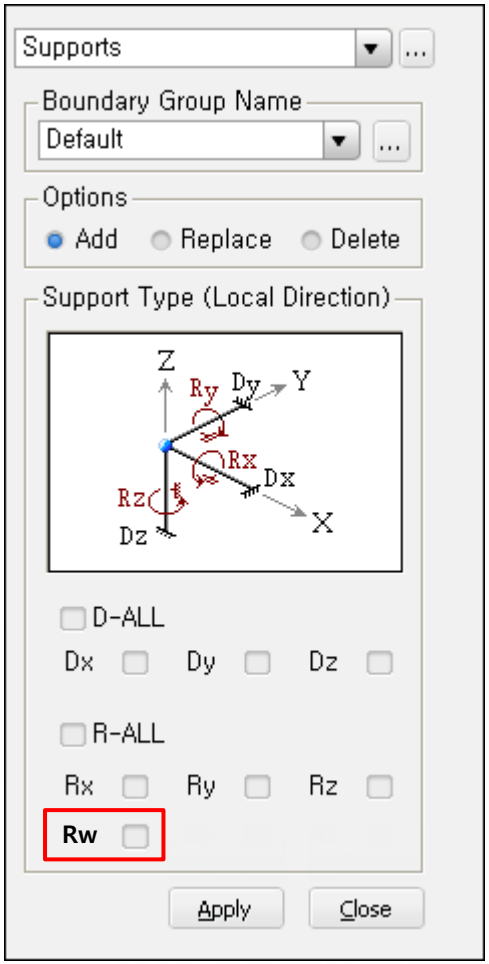
CONSIDER WARPING EFFECT



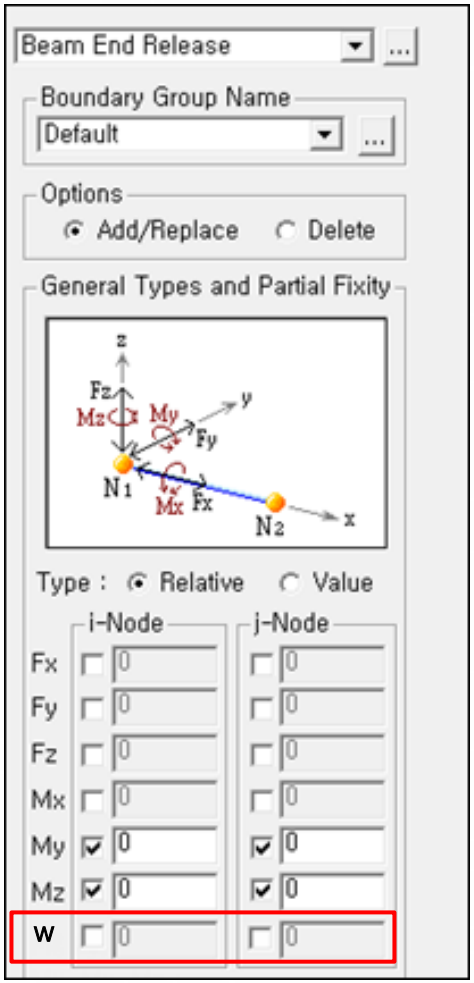
➢ 7th DOF is applicable to "General beam/Tapered beam".

CONSIDER WARPING EFFECT IN BOUNDARY CONDITIONS

< Support >

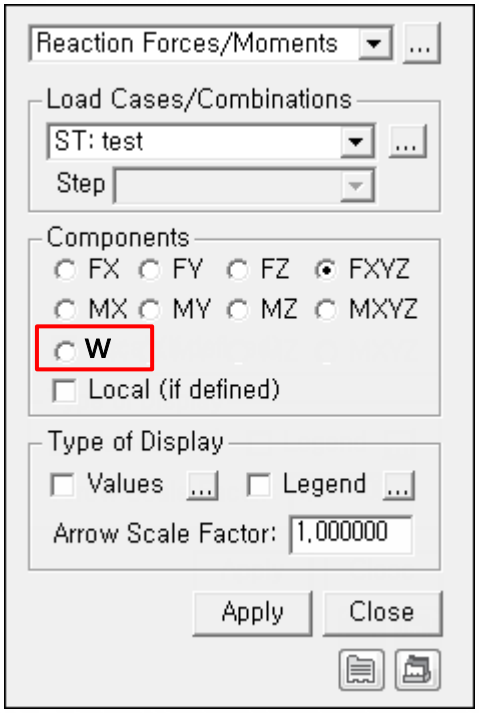


< Beam End Release >

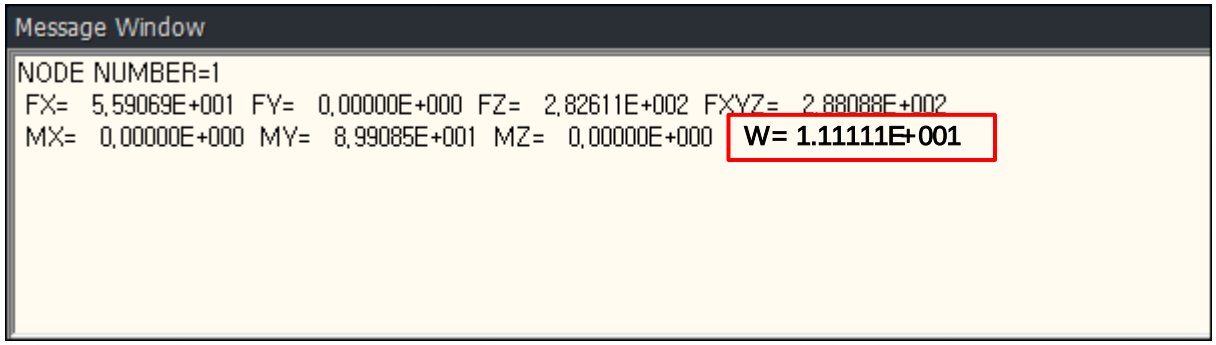


CONSIDER WARPING EFFECT IN REACTIONS

< Reaction Forces/Moments >

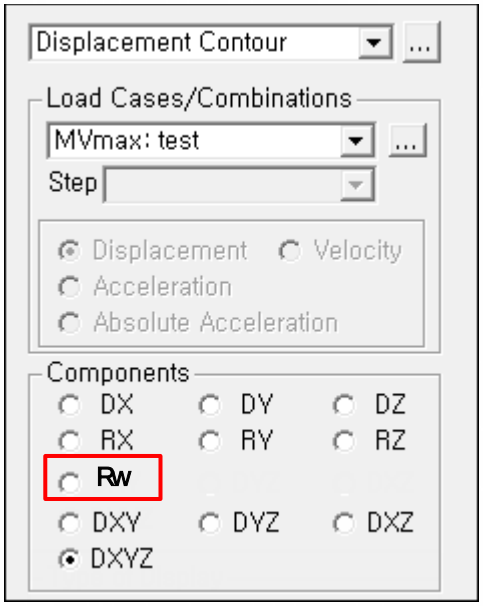


< Search Reaction Forces/Moments >



CONSIDER WARPING EFFECT - DEFORMATION

< Displacement Contour >



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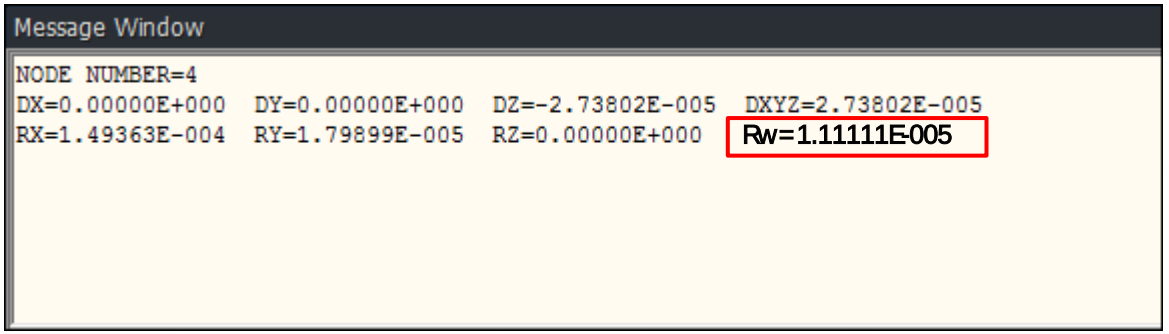
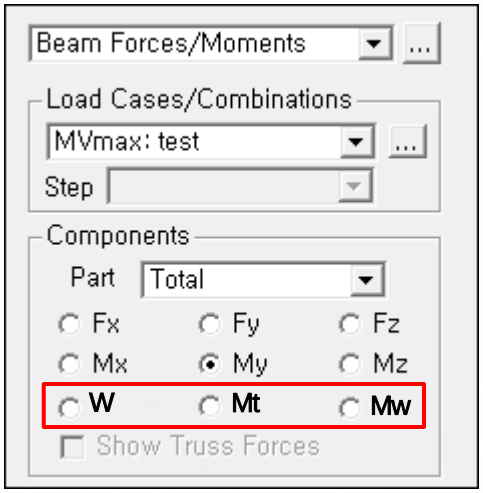


CONSIDER WARPING EFFECT - DEFORMATION

< Beam Forces/Moments >
< Beam Diagrams >



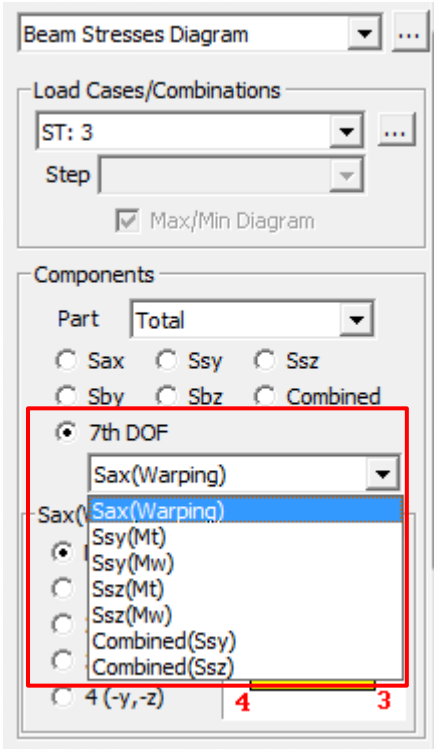
➢ **Component**

- W (Bi-Moment)
- Mt (Twisting Moment)
- Mw (Warping Moment)

Unit: W (Bi-Moment) – N*m^2

CONSIDER WARPING EFFECT - STRESSES

< Beam Stresses >
< Beam Stresses Diagram >

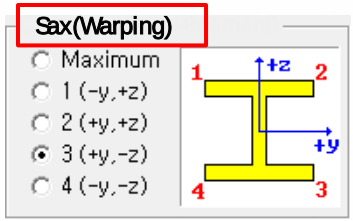


➢ **Component "7th DOF"**

- Sax(Warping) : Warping Normal Stress
- Ssy(Mt) : Twisting Moment Shear Stress(τxy)
- Ssy(Mw) : Warping Moment Shear Stress(τxy)
- Ssz(Mt) : Twisting Moment Shear Stress(τxz)
- Ssz(Mw) : Warping Moment Shear Stress(τxz)
- Combined(Ssy) : "Transverse + Twisting Moment + Warping Moment" Shear Stress(τxy)
- Combined(Ssz) : "Transverse + Twisting Moment + Warping Moment" Shear Stress(τxz)

※ **"Combied(Normal) : Axial + Moment + Warping"** when Combined is selected
- W/O warping= Axial + Moment
- With Warping = Axial + Moment + Warping

- ex) Sax(Warping:



CONSIDER WARPING EFFECT - TABLE

< Reaction Table >

	Node	Load	FX (kN)	FY (kN)	FZ (kN)	MX (kN·m)	MY (kN·m)	MZ (kN·m)	W (kN·m)
▶	1	test	0.000000	0.000000	100.000000	-100.000000	-500.000000	0.000000	0.000000
	SUMMATION OF REACTION FORCES PRINTOUT								
		Load	FX (kN)	FY (kN)	FZ (kN)				
		test	0.000000	0.000000	100.000000				

< Displacement Table >

	Node	Load	DX (m)	DY (m)	DZ (m)	RX ([rad])	RY ([rad])	RZ ([rad])	Rw ([rad])
▶	1	test	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	2	test	0.000000	0.000000	-0.000113	0.000373	0.000028	0.000000	0.000000
	3	test	0.000000	0.000000	-0.000009	0.000075	0.000010	0.000000	0.000000
	4	test	0.000000	0.000000	-0.000027	0.000149	0.000018	0.000000	0.000000
	5	test	0.000000	0.000000	-0.000052	0.000224	0.000024	0.000000	0.000000
	6	test	0.000000	0.000000	-0.000082	0.000299	0.000027	0.000000	0.000000

Twisting Moment

Warping Moment

< Beam Force Table >

	Elem	Load	Part	Axial (kN)	Shear-y (kN)	Shear-z (kN)	Torsion (kN·m)	Moment-y (kN·m)	Moment-z (kN·m)	Bi-Moment (kN·m)	T-Moment (kN·m)	W-Moment (kN·m)
	1	test	I[1]	0.00	0.00	-100.00	100.00	-500.00	0.00	100.00	-500.00	0.00
	1	test	J[3]	0.00	0.00	-100.00	100.00	-400.00	0.00	100.00	-400.00	0.00
	2	test	I[3]	0.00	0.00	-100.00	100.00	-400.00	0.00	100.00	-400.00	0.00
	2	test	J[4]	0.00	0.00	-100.00	100.00	-300.00	0.00	100.00	-300.00	0.00

CONSIDER WARPING EFFECT - TABLE

< Beam Stress(7th DOF) Table >, < Beam Stress(7th DOF)(PSC) Table >

	Elem	Load	Part	Section Position	Sax(Warping) (kN/m^2)	Ssy(Mt) (kN/m^2)	Ssy(Mw) (kN/m^2)	Ssz(Mt) (kN/m^2)	Ssz(Mw) (kN/m^2)	Cb(Ssy) (kN/m^2)	Cb(Ssz) (kN/m^2)
▶	1	1	I[1]	Pos-1	6.78e+007	0.00e+000	4.53e+005	0.00e+000	1.23e+005	4.53e+005	3.17e+004
	1	1	J[2]	Pos-1	3.51e+007	-1.24e+007	2.35e+005	-6.85e+006	6.39e+004	-1.21e+007	-6.87e+006
	2	1	I[2]	Pos-1	3.51e+007	-1.24e+007	2.35e+005	-6.85e+006	6.39e+004	-1.21e+007	-6.87e+006
	2	1	J[3]	Pos-1	1.82e+007	-1.88e+007	1.22e+005	-1.04e+007	3.31e+004	-1.87e+007	-1.04e+007
	3	1	I[3]	Pos-1	1.82e+007	-1.88e+007	1.22e+005	-1.04e+007	3.31e+004	-1.87e+007	-1.04e+007
	3	1	J[4]	Pos-1	9.41e+006	-2.21e+007	6.29e+004	-1.22e+007	1.71e+004	-2.20e+007	-1.23e+007
	4	1	I[4]	Pos-1	9.41e+006	-2.21e+007	6.29e+004	-1.22e+007	1.71e+004	-2.20e+007	-1.23e+007
	4	1	J[5]	Pos-1	4.87e+006	-2.38e+007	3.26e+004	-1.32e+007	8.87e+003	-2.38e+007	-1.33e+007
	5	1	I[5]	Pos-1	4.87e+006	-2.38e+007	3.26e+004	-1.32e+007	8.87e+003	-2.38e+007	-1.33e+007
	5	1	J[6]	Pos-1	2.52e+006	-2.47e+007	1.69e+004	-1.37e+007	4.59e+003	-2.47e+007	-1.38e+007
	6	1	I[6]	Pos-1	2.52e+006	-2.47e+007	1.69e+004	-1.37e+007	4.59e+003	-2.47e+007	-1.38e+007
	6	1	J[7]	Pos-1	1.30e+006	-2.52e+007	8.77e+003	-1.39e+007	2.39e+003	-2.51e+007	-1.40e+007
	7	1	I[7]	Pos-1	1.30e+006	-2.52e+007	8.77e+003	-1.39e+007	2.39e+003	-2.51e+007	-1.40e+007
	7	1	J[8]	Pos-1	6.63e+005	-2.54e+007	4.60e+003	-1.41e+007	1.25e+003	-2.54e+007	-1.41e+007
	8	1	I[8]	Pos-1	6.63e+005	-2.54e+007	4.60e+003	-1.41e+007	1.25e+003	-2.54e+007	-1.41e+007
	8	1	J[9]	Pos-1	3.25e+005	-2.55e+007	2.51e+003	-1.41e+007	6.82e+002	-2.55e+007	-1.42e+007
	9	1	I[9]	Pos-1	3.25e+005	-2.55e+007	2.51e+003	-1.41e+007	6.82e+002	-2.55e+007	-1.42e+007
	9	1	J[10]	Pos-1	1.33e+005	-2.56e+007	1.53e+003	-1.41e+007	4.18e+002	-2.56e+007	-1.42e+007

APPLICABLE ANALYSIS TYPE

< 1. Static Analysis>

<2. Eigenvalue Analysis >

- Results Tables > Vibration Mode Shape

< 3. Response Spectrum Analysis>

- Results > Nodal Result of RS
- Results Tables > Nodal Result of RS

< 4. Construction Stage Analysis >

- Results > Bridge Girder Diagram
- Results Tables > Composite Section for C.S > Beam Force > Beam Stress
- Results Tables > Construction Stage > Beam Section Properties at Last Stage